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Three-dimensional elasticity solution for sandwich panels with corrugated cores by using energy method

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ABSTRACT

Sandwich panels having metallic corrugated cores extremely utilized in various engineering fields such as aerospace, automotive, civil, and naval engineering. Although significant efforts have been dedicated in research into corrugated sandwich panels, analytical solutions are still very few. In this paper, analysis of corrugated sandwich panels with trapezoidal shape is carried out analytically.

In the present study, a new analytical approach is developed for bending analysis of corrugated sandwich panels which have not been considered yet. The out-of-plane properties of corrugated core are obtained based on three dimensional theory of elasticity. The state-space method is implemented in conjunction with double Fourier series to solve the coupled partial differential equations. To validate the present approach, numerical results are compared with the results reported in the literature. The sensitivity analysis of the stress and displacement components to the geometrical parameters of corrugated core and its orthotropy is presented by introducing dimensionless ratios.

1. Introduction

Metallic sandwich panels are one group of laminated composites which are used extensively due to their high stiffness and light weight. These panels consist of thin facings sandwiching a core. The high thickness of core provides higher moment of inertia and increase bending stiffness of sandwich panel. Corrugated sandwich panels are a special type of sandwich panels which have corrugated metallic core with alternate ridges and grooves shaped. These types of panels have very encouraging commercial advantages and they are widely used in various areas such as aerospace, automotive, civil, and naval engineering. Because corrugated cores have complex shaped, equivalent material models are required to obtain equivalent properties of them.

Many researchers studied the equivalent properties of corrugated cores. One of the early works is the work of Libove and Hubka [1]. They showed that shear effects is not negligible and should be considered in equivalent orthotropic plates. Chang et al. [2] computed equivalent in-plane properties of corrugated sandwich plates and investigated the bending behavior of corrugated sandwich panels by using Mindlin–Reissner plate theory. By considering classical plate theory (CPT), Xia et al. [3] used equivalent force method to derive equal stiffness terms for any corrugation shapes.

Based on Mindlin–Reissner theory, Bartolozzi et al. [4] studied acoustic behavior of corrugated core by using energy method to derive equivalent in-plane properties of corrugated core with sinusoidal shape. In other work, they extended the energy method to characterize the equivalent method for arbitrary shape of corrugated cores [5]. Magnucka-Blandzi et al. [6,7] used classical Euler–Bernoulli and broken–-line hypothesis to analyze bending and buckling behavior of sandwich beams with sinusoidal corrugated cores. They also studied the effect of transverse shearing deformation for short and long beams.

Based on first-order shear deformation theory (FSDT), Peng et al. [8] investigated free vibration analysis of corrugated-core sandwich plates with orthotropic cores by using mesh-free Galerkin method. The axial crushing responses of multi-layer trapezoidal aluminum corrugated core sandwich structures, were investigated at quasi-static and dynamic strain rates by Cenk et al. [9].

Zheng et al. [10] proposed an equivalent plate model for corrugated cores by using classical shell theory. They presented a complete set of effective in-plate stiffness. Park et al. [11] extended the homogenization model for corrugated composite cores and presented explicit expressions to calculate effective extensional and bending stiffness for them. They showed that effective stiffness and the anisotropy considerably is affected by ply angles. Buannic et al. [12] considered unit cell made of two thin faces with corrugated core and obtained the effective properties of sandwich panels. They determined the deflection

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Nomenclature

A	cross section
A	length
B	width
C_{ij}	equivalent constants
c_{ij}	stiffness elastic constants
E	elastic modulus
G	shear modulus
h_c	height of core
h_f	facing thickness
h_t	total height of panel
I	moment of inertia
L	Length
M_O	external moment
M	bending moment
N	internal normal force
P	half period of core
Q	external distributed force
T	internal tangential force
t_c	thickness of core sheet
u, v, w	displacement components in the x, y and z
$\overline{U}, \overline{V}, \overline{W}$	non-dimensional displacement
V	external vertical force
x, y, z	Cartesian reference coordinates, where x is along the corrugation direction

Greek symbols

γ_{ij}	shear strain
δ_H	horizontal displacement
δ_V	vertical displacement
E	normal strain
H	non-dimensional thickness coordinate
ν_{zx}	Poisson's ration
σ_i	normal stress
τ_{ij}	shear stress
φ	angle of tangent line

Subscripts

0	z = 0
n, m	half wave numbers in the x and y directions
xyz	reference coordinates

of sandwich panels by using Kirchhoff–Love theory. But they acknowledged that it is necessary to use complicated theories like Mindlin–Reissner models to provide accurate results for stiffer panels. Yan et al. [13] presented analytical formulas based on three-point bending test and compared their results with experimental results. Martinez et al. [14] determined equivalent plate model for composite corrugated-core sandwich panels. Boorle and Mallick [15] extended micromechanical approach used by Martinez et al. [14] to consider effects of various geometric parameters on bending response of composite corrugated sandwich panels. Walczak et al. [16] studied Buckling and vibration of metal sandwich beams with trapezoidal corrugated cores. They used Hamilton's principle in conjunction with Kirchhoff-Love hypothesis to derive the governing equations for three-layered corrugated beams.

Although analytical and numerical treatments of corrugated sandwich panels have been performed by many researches but most of them have been limited to calculate the equivalent in-plane properties of corrugated sandwich panels, despite the fact that corrugated sandwich panels are three-dimensional structures and as a result of those extreme orthotropic natures, equivalent out-of-plane cannot be ignored. The current study extends the work by Bartolozzi et al. [4] for out-of-plane properties of corrugated sandwich panels. Moreover, the three dimensional bending analysis are conducted by considering the obtained equivalent out-of-plane properties. The three dimensional coupled partial differential equations are reduced to the ordinary differential equations by expanding the field variables to double Fourier series along in-plane directions. Then, the state space method is implemented along the thickness direction to solve the problem analytically. The obtained results are compared with classical plate theory (CPT) and the reliability of the CPT is examined.

2. Equivalent orthotropic properties in the z-direction

In this study, trapezoidal sandwich panels are considered with overall thickness h_t, length a and width b as shown in Fig. 1. According to the figure, trapezoidal sandwich panel made up of two upper and lower facing sheets with thickness h_f and a thick core with height of h_c.

The corrugated sandwich panel is considered as a multi-layer composite plate. Using geometrical homogenization approach, the equivalent out-of-plane parameters, i.e. elastic modulus, E_z and Poisson's ration, ν_{zx} are computed. Without loss of generality, trapezoidal shape is considered for corrugated core in the *x-direction*. Fig. 2(a) shows schematically a unit-cell of corrugated core contains unit cell of trapezoidal shape. The unit-cell is originated at the lowest clamped end point and the width of the core is assumed to be unit, $b = 1$. As represented in Fig. 2(b), by imposing a vertical force, V, in the *z-direction* to the upper end of trapezoidal shape, positive vertical displacement, δ_V and negative horizontal displacement in the other direction, δ_H occurs. It is worthy to note that a dummy moment, M_0, should be applied to the upper end of unit cell to avoid rotation in the *xz-plane*.

The produced bending moment, normal and tangential forces at a distance x, are

$$M(x) = V\left(\frac{c}{2} - x\right) - M_0 \tag{1-a}$$

$$N(x) = \begin{cases} -V\sin\theta & (AB) \\ 0 & (BC) \end{cases} \tag{1-b}$$

$$T(x) = \begin{cases} V\cos\theta & (AB) \\ V & (BC) \end{cases} \tag{1-c}$$

where θ is angle of tangent. Applying the Castigliano's second theorem, the rotation of the upper end and its vertical and horizontal displacements can be derived as follow:

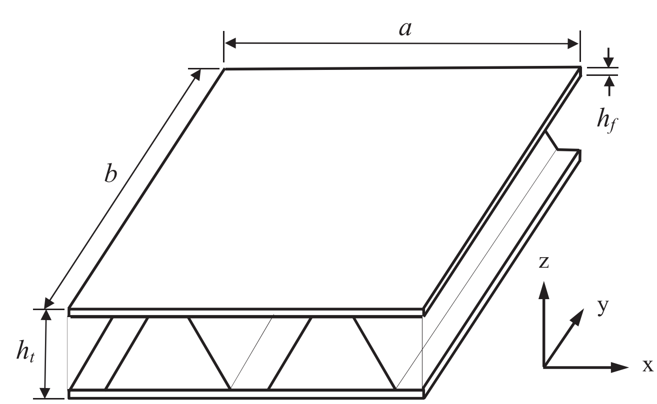

Fig. 1. Schematic representation of sandwich panel with trapezoidal corrugated core.

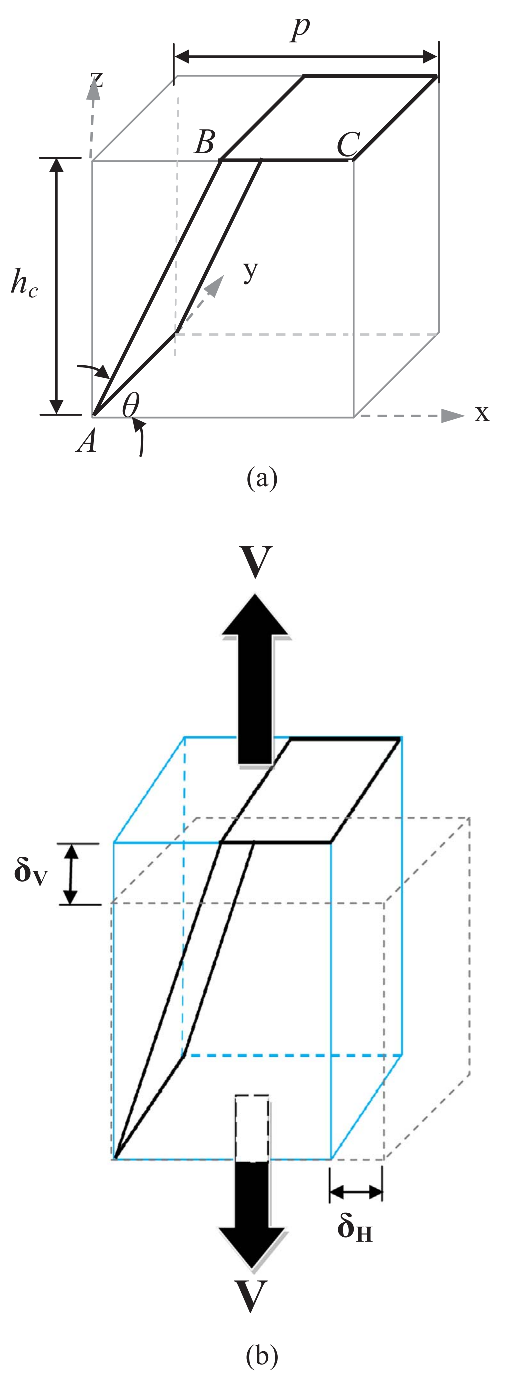

Fig. 2. A unit-cell of sinusoidal corrugated core (a) before deformation (b) deformation orthogonal to the corrugation direction.

$$\delta_{M_0} = \int_0^p \left(\frac{M}{EI}\frac{\partial M}{\partial M_0} + \frac{N}{EA}\frac{\partial N}{\partial M_0} + \frac{T}{GA'}\frac{\partial T}{\partial M_0}\right)\frac{dx}{\cos\theta}. \tag{2-a}$$

$$\delta_V = \int_0^p \left(\frac{M}{EI}\frac{\partial M}{\partial V} + \frac{N}{EA}\frac{\partial N}{\partial V} + \frac{T}{GA'}\frac{\partial T}{\partial V}\right)\frac{dx}{\cos\theta} \tag{2-b}$$

$$\delta_H = \int_0^p \left(\frac{M}{EI}\frac{\partial M}{\partial H} + \frac{N}{EA}\frac{\partial N}{\partial H} + \frac{T}{GA'}\frac{\partial T}{\partial H}\right)\frac{dx}{\cos\theta} \tag{2-c}$$

where A and I are the area and moment of inertia of the core cross-section. A' is $\varkappa A$ where $\varkappa$ is the shear correction factor ($\varkappa = 5/6$). Solving Eqs. (2-a) and (2-b) by considering two conditions $V = 1$ and $\delta_{M_0} = 0$ and then substituting the results in Eq. (2-c), results in the following relations

$$\delta_V = C_{22} - \frac{C_{23}^2}{C_{33}} \tag{3-a}$$

$$\delta_H = C_{12} - \frac{C_{13}C_{23}}{C_{33}} \tag{3-b}$$

The equivalent elastic modulus in the *z-direction* is then as follow

$$E_z = \frac{\sigma_z}{\varepsilon_z} = (F_z/A_{xy})/(\delta_V/l_z) = \frac{2h_c}{p\,\delta_V} \tag{4}$$

To obtain the Poisson's ratio, ν_{zx}, one should calculate the ratio of lateral strain along *x-direction* and the axial strain along *z-direction*. Thus

$$\nu_{zx} = -\frac{\varepsilon_x}{\varepsilon_z} = (\delta_H/p)/(\delta_V/2h_c) = \frac{2h_c}{p}\left[C_{12} - \frac{C_{13}C_{23}}{C_{33}}\right]\Big/\left[C_{22} - \frac{C_{23}^2}{C_{33}}\right] \tag{5}$$

where C_{ij} is a symmetric matrix introduced in Appendix.

3. Governing equations

By considering the sandwich panel as laminated composite panel consist of two facing sheets and equivalent thick orthotropic core layer, for each layer the equilibrium equations are as follow:

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} = 0,$$
$$\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} = 0,$$
$$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} = 0 \tag{6}$$

The constitutive equations for an orthotropic layer are:

$$\{\sigma\} = [c]\{\varepsilon\} \tag{7}$$

where $\{\sigma\} = \{\sigma_x \ \ \sigma_y \ \ \sigma_z \ \ \tau_{zy} \ \ \tau_{zx} \ \ \tau_{xy}\}^T$, $\{\varepsilon\} = \{\varepsilon_x \ \ \varepsilon_y \ \ \varepsilon_z \ \ \gamma_{zy} \ \ \gamma_{zx} \ \ \gamma_{xy}\}^T$. $[c]$ is material matrix as follow

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \tag{8}$$

It is clear that, for corrugated core, the equivalent three-dimensional properties are used in the above matrix; while for top and bottom faces only two independent isotropic engineering constants, i.e. E and ν are used. The linear strain-displacement relations are given by

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \gamma_{zy} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$
$$\varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$
$$\varepsilon_z = \frac{\partial w}{\partial z} \qquad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{9}$$

Mechanical boundary conditions for outer surfaces of sandwich panel are considered as follow

$$\sigma_z = \tau_{zx} = \tau_{zy} = 0 \qquad at \quad z = 0$$
$$\sigma_z = q = q^*.\ \sin\left(\frac{\pi x}{a}\right).\ sin\left(\frac{\pi y}{b}\right), \quad \tau_{zx} = \tau_{zy} = 0 \quad at \quad z = h \tag{10}$$

4. Exact solution for stress field

For a corrugated sandwich panels with four edges simply supported, following boundary conditions should be satisfied;

$$u = w = 0, \quad \sigma_y = 0 \quad at \quad y = 0, \ b$$
$$v = w = 0, \quad \sigma_x = 0 \quad at \quad x = 0, \ a \tag{11}$$

Following Fourier series expansion of stress and displacement fields satisfy relations for simply supported boundary conditions

$$\begin{aligned}
u(x, y, z) &= u^*.\ \cos(q_n x).\ \sin(p_m y)\\
v(x, y, z) &= v^*.\ \sin(q_n x).\ \cos(p_m y)\\
w(x, y, z) &= w^*.\ \sin(q_n x).\ sin(p_m y)\\
\sigma_x(x, y, z) &= \sigma_x^*.\ \sin(q_n x).\ \sin(p_m y)\\
\sigma_y(x, y, z) &= \sigma_y^*.\ \sin(q_n x).\ sin(p_m y)\\
\sigma_z(x, y, z) &= \sigma_z^*.\ \sin(q_n x).\ sin(p_m y)\\
\tau_{zy}(x, y, z) &= \tau_{zy}^*.\ \sin(q_n x).\ \cos(p_m y)\\
\tau_{zx}(x, y, z) &= \tau_{zx}^*.\ \cos(q_n x).\ sin(p_m y)\\
\tau_{xy}(x, y, z) &= \tau_{xy}^*.\ \cos(q_n x).\ cos(p_m y)
\end{aligned} \tag{12}$$

where $p_m = \frac{m\pi}{b}$, $q_n = \frac{n\pi}{a}$ and u^*, v^*, w^*, σ_x^*, σ_y^*, σ_z^*, τ_{zy}^*, τ_{zx}^*, τ_{xy}^* are functions of z. By substitution of relations (12) into Eqs. (6) and (7), state-space equations are derived as follow:

$$\frac{d}{dz}\{\delta\} = G\{\delta\} \tag{13}$$

where $\{\delta\} = \{\sigma_z^*\ \ u^*\ \ v^*\ \ w^*\ \ \tau_{zx}^*\ \ \tau_{zy}^*\}^T$ and G is the coefficients matrix (see Appendix). The general solution to Eq. (13) explicitly expressed as

$$\{\delta\} = e^{\int_0^z G.dz}\{\delta_0\} = e^{G.z}\{\delta_0\} \tag{14}$$

where $\{\delta_0\}$ is the state vector at the lowest surface. Imposing surface boundary conditions at the upper and lower surface, Eq. (10), the following equations can be obtained:

$$\begin{bmatrix} [A] \\ [S]^H \end{bmatrix}\{\delta_0\} = \{0\ \ 0\ \ 0\ \ -q\ \ 0\ \ 0\}^T \tag{15}$$

where matrix $[S]^H$ is $e^{[G].h}$ and $[A]$ is given in Appendix. State vector at the bottom surface, $\{\delta_0\}$, is obtained by solving Eq. (15). Finally substitution of obtained $\{\delta_0\}$ into Eq. (14) yields the state variables in the *z-direction.*

5. Results and discussion

To validate the present formulation and ensure the accuracy of the proposed solution method, the obtained results compared with works done by Bartolozzi [4], Magnucka-Blandzi [7] and Briassoulis [17]. Good agreement is evident between results as shown in Table 1.

Based on industrial applications, the following initial values are considered in the parametric studies: Young's modulus: $E = 71$ GPa, Poisson's ratio: $\nu = 0.33$, length and width: $a = b = 1$ m, core and facing thickness: $t_c = h_f = 0.8$ mm, and pitch $p = 7.5$ mm.

Fig. 3 shows four equivalent out-of-plane mechanical constants as a function of the non-dimensional ratios p/h_c and h_c/t_c. It is obvious that for these cases, the equivalent properties of the core changes nonlinearly. Indeed, E_z, G_{yz} and G_{xz} decrease when the ratio of either p/h_c or h_c/t_c increase, but the effect of h_c/t_c is more notable. On the other hand, under constant p/h_c ratio, the decrease of core sheet thickness, t_c to height of core h_c (i.e. h_c/t_c) declines all equivalent out-of- plane modulus in the *z-direction* and make the core softened. On contrast, it can be concluded that Poisson's ratio, ν_{xz} does not affected by the h_c/t_c ratios. The influence of the corrugated shapes on the in-plane and transverse displacements is also investigated. For simplicity, non-dimensional displacements and stresses are defined as follow:

$$\overline{U} = \frac{u^*}{h},\quad \overline{V} = \frac{v^*}{h},\quad \overline{W} = \frac{w^*}{h},\quad \overline{\sigma}_i = \frac{\sigma_i^*}{q_0},\quad \overline{\tau}_{ij} = \frac{\tau_{ij}^*}{q_0}$$

Fig. 4 shows the through-thickness distributions of the dimensionless displacements, $\overline{U}$, $\overline{V}$, $\overline{W}$ with different ratios of h_c/t_c. The x-axis represents the non-dimensional thickness coordinate ($\eta = z/h_t - 0.5$). In contrast to transverse displacement, $\overline{W}$, the magnitude of the in-plane displacements $\overline{U}$ and $\overline{V}$ is found to be higher for higher h_c/t_c ratios. But the in-plane displacements across the thickness are very small and negligible in comparison to transverse displacement.

The nonlinear through-thickness distributions of normal and shear stresses $\overline{\sigma}_z$, $\overline{\tau}_{xz}$, $\overline{\tau}_{yz}$ are shown in Fig. 5. Non-dimensional stresses satisfy the top and bottom surface boundary conditions declared in Eq. (10). Clearly, all the variations through the thickness are nonlinear and the nonlinearity is more remarkable in higher ratios of h_c/t_c especially for shear stress. The maximum value of $\overline{\tau}_{zx}$, $\overline{\tau}_{zy}$ increased as the ratio of h_c/t_c decrease. It can be concluded that due to the symmetry of loading, the shear stress distributions has symmetry with respect to the mid-surface of the panel.

As shown in Figs. 4 and 5, the presented method guarantees the continuity of all displacements and stresses, but not the smoothness of them, because the slope of curves is not continuous. The only exception is transverse displacement, $\overline{W}$. It is concluded that sudden change in the slope of curves occurs at the interface between the core and facing sheets. This is due to the fact that equivalent elastic constants of corrugated core and facing sheets are dissimilar which constitute discontinuity in the curves slope. Effect of t_c/t_f together with h_c/t_c on the non-dimensional transverse displacement, $\overline{W}$ is presented in Fig. 6. According to the figure and as expected, transverse displacement increase when the thickness of facing decrease. Moreover, comparing the elasticity results with results of CPT reveal that classical plate theory underestimates center deflection of sandwich panel. In fact, classical plate theory considered the three-dimensional plate in two-dimensional form and the properties and behavior in the thickness direction cannot be captured. So the error caused by this conventional theory is significant and cannot be ignored especially when the ratio of h_c/t_c is small.

6. Conclusion

This paper presents an analytical formulation for trapezoidal corrugated sandwich panels based on three dimensional elasticity theory. The geometrical homogenization approach is used to obtain the main equivalent out-of-plane parameters. The equations of non-

Table 1
Comparison of bending stiffness for sinusoidal corrugated core ($E = 71000$ MPa, $\nu = 0.33$, $t_c = 0.3$ mm, $h_c = 4$ mm, $p = 4.25$ mm, $b = 1$ m).

p (mm)	**t_c (mm)**	**G_{yz} (MPa)**			**E_y (GPa)**			**E_x (MPa)**			
		Present	***Ref.* [4]**	***Ref.* [7]**	***Present***	***Ref.* [4]**	***Ref.* [7]**	***Present***	***Ref.* [4]**	***Ref.* [7]**	***Ref.* [17]**
4	**0.15**	683.85	656.40	697.11	3.90	3.72	4.57	1.99	1.92	2.61	1.79
	0.30	1367.71	1313.20	1421.34	7.79	7.43	8.33	15.81	15.13	21.27	14.25
	0.45	2051.56	1969.30	2173.59	11.69	11.21	11.36	52.75	50.62	73.18	47.30
6	**0.15**	360.43	346.06	364.93	3.29	3.13	4.12	2.22	2.11	3.07	2.01
	0.30	720.86	692.04	738.87	6.57	6.36	7.56	17.65	16.91	24.87	15.50
	0.45	1081.29	1038.07	1121.82	9.86	9.48	10.37	59.05	56.62	84.98	53.01
8	**0.15**	219.54	210.73	221.38	3.03	2.90	3.95	2.32	2.27	3.31	2.07
	0.30	439.07	421.53	446.40	6.07	5.85	7.26	18.51	17.72	26.72	15.90
	0.45	658.61	632.25	674.98	9.10	8.72	9.98	62.01	59.51	90.89	55.64

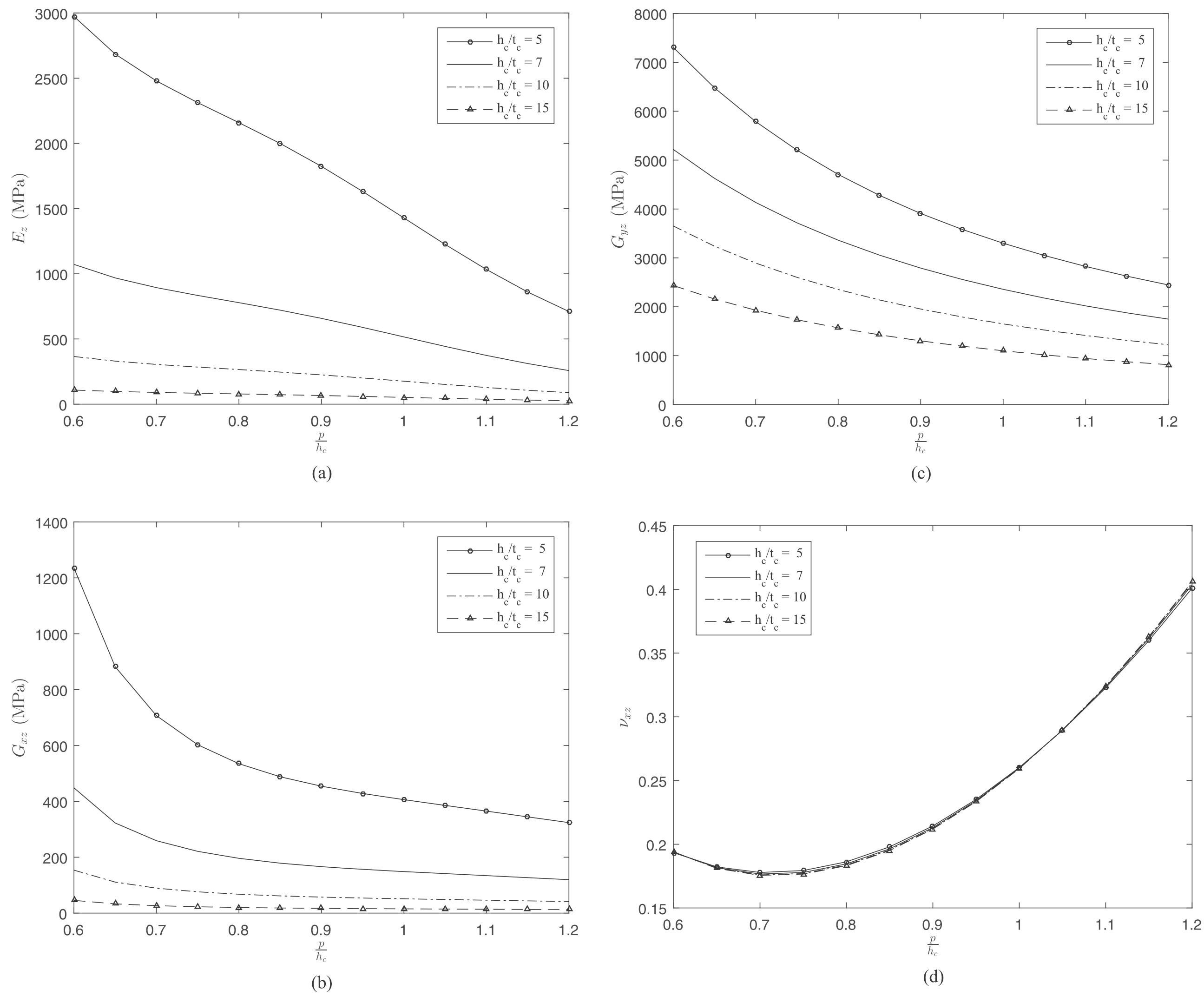

Fig. 3. Out-of-plane equivalent elastic constants of core against non-dimensional ratio p/h_c for various h_c/t_c ratios ($t_c = 0.8$ mm, $t_c/t_f = 1$, $\theta = 63.4°$).

homogeneous, orthotropic laminated plates with equivalent properties for corrugated core are derived and put in a state-space matrix. To obtain a closed form solution for the corrugated sandwich panel, the shape functions are used through the in-plane directions. Then the state-space method was used to solve the resulting equations along the thickness direction. It was found that elastic modulus in the thickness direction weakens when increasing pitch and height of corrugations, and strengthens when sheet of core becomes thicker. The through-thickness distributions of stress and displacement fields are presented and the influences of dimensionless geometrical quantities are investigated. By using the present method, inter-laminar stresses between corrugated core and facings are obtained and the layer debonding can predict. The classical plate theory are compared with obtained results and it is found that classical plate theory is not reliable for thick sandwich panel and cannot capture the properties in the thickness direction.

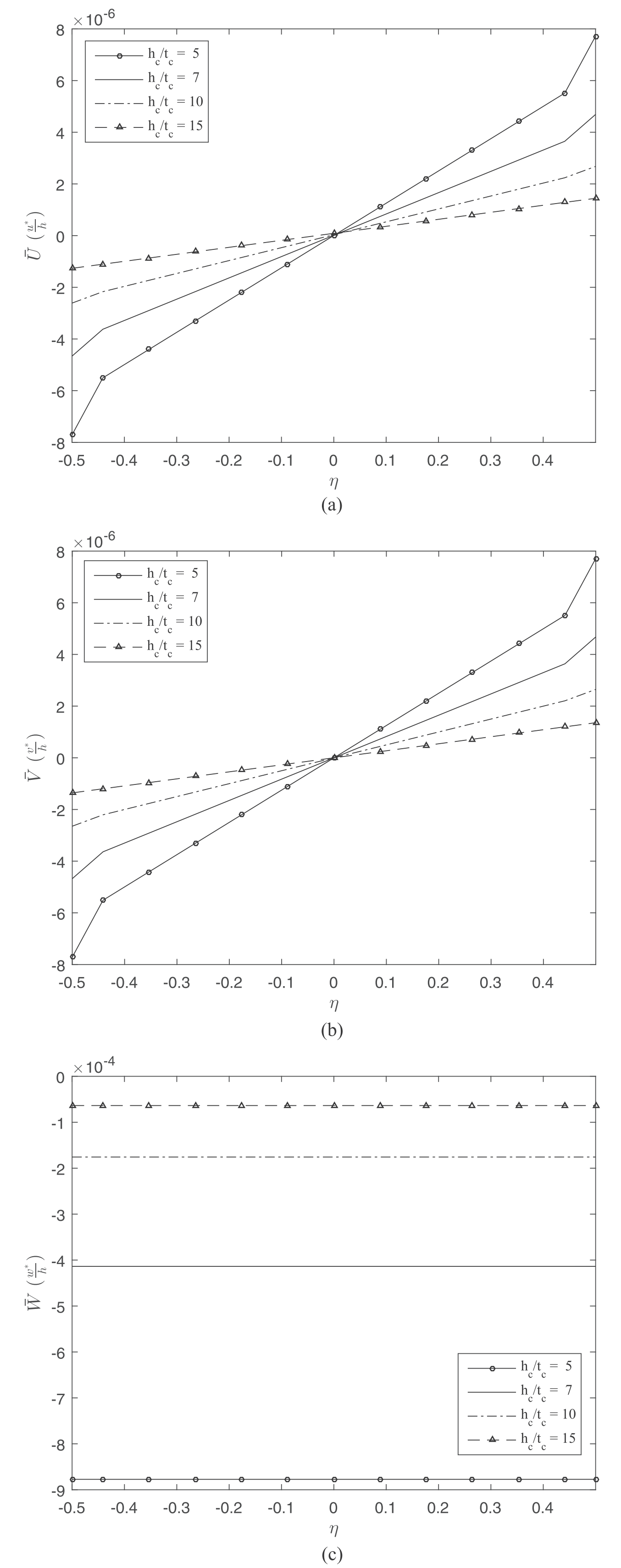

Fig. 4. Influence of non-dimensional ratio h_c/t_c on the displacements of corrugated sandwich panels ($t_c = 0.8$ mm, $t_c/t_f = 1, p/h_c = 0.6, \theta = 63.4°$).

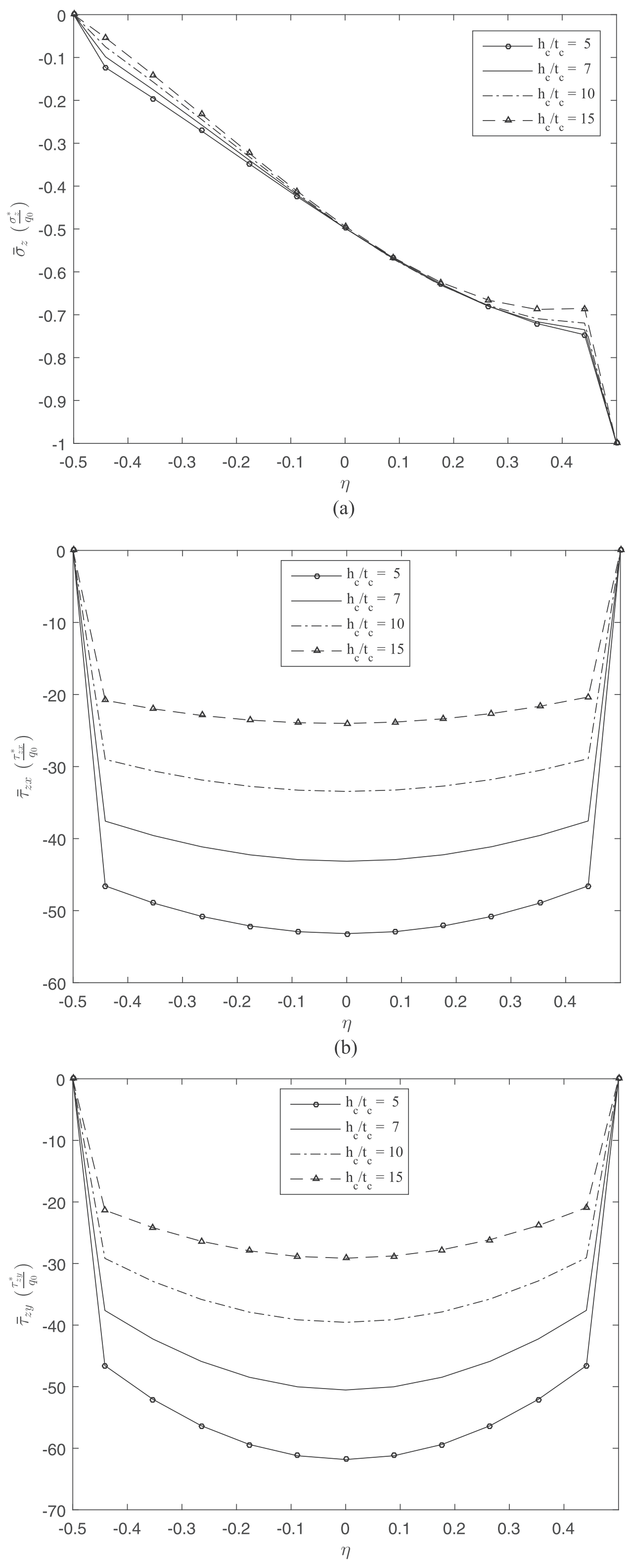

Fig. 5. Influence of non-dimensional ratio h_c/t_c on the stresses of corrugated sandwich panels ($t_c = 0.8$ mm, $t_c/t_f = 1, p/h_c = 0.6, \theta = 63.4°$).

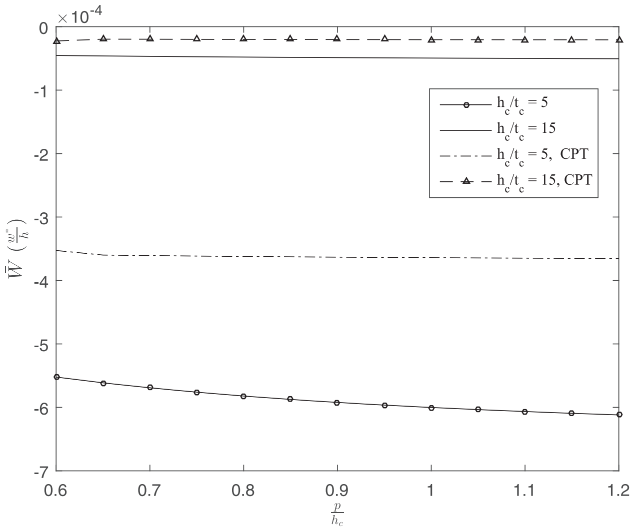

Fig. 6. Transverse displacement of sandwich panel against non-dimensional ratio p/h_c for various h_c/t_c ratios ($t_c = 0.8$ mm, $x = a/2$, $y = b/2$, $z = h/2$, $t_c/t_f = 1$).

Appendix

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ sym. & & C_{33} \end{bmatrix}$$

$$C_{11} = \frac{12}{t_c^2}\int_0^p f^2\sqrt{1+f'^2}\,dx + \int_0^p \frac{1}{\sqrt{1+f'^2}}dx + \frac{2(1+\nu)}{5/6}\int_0^p \frac{f'^2}{\sqrt{1+f'^2}}dx$$

$$C_{12} = \frac{12}{t_c^2}\int_0^p x f\sqrt{1+f'^2}\,dx + \left(\frac{2(1+\nu)}{5/6} - 1\right)\int_0^p \frac{f'}{\sqrt{1+f'^2}}dx$$

$$C_{13} = -\frac{12}{t_c^2}\int_0^p f\sqrt{1+f'^2}\,dx$$

$$C_{22} = \frac{12}{t_c^2}\int_0^p x^2\sqrt{1+f'^2}\,dx + \int_0^p \frac{f'^2}{\sqrt{1+f'^2}}dx + \frac{2(1+\nu)}{5/6}\int_0^p \frac{1}{\sqrt{1+f'^2}}dx$$

$$C_{23} = -\frac{12}{t_c^2}\int_0^p x\sqrt{1+f'^2}\,dx$$

$$C_{33} = -\frac{12}{t_c^2}\int_0^p \sqrt{1+f'^2}\,dx$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & q & p \\ 0 & 0 & 0 & -q & 1/c_{55} & 0 \\ 0 & 0 & 0 & -p & 0 & 1/c_{44} \\ 1/c_{33} & qc_{13}/c_{33} & pc_{23}/c_{33} & 0 & 0 & 0 \\ -qc_{13}/c_{33} & G_{52} & G_{53} & 0 & 0 & 0 \\ -pc_{23}/c_{33} & G_{62} & G_{63} & 0 & 0 & 0 \end{bmatrix}$$

where:

$$G_{52} = q^2\left(c_{11} - \frac{c_{13}^2}{c_{33}}\right) + p^2 c_{66}$$

$$G_{53} = G_{62} = pq\left(c_{12} - \frac{c_{13}c_{23}}{c_{33}} + c_{66}\right)$$

$$G_{62} = p^2\left(c_{22} - \frac{c_{23}^2}{c_{33}}\right) + q^2 c_{66}$$

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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